DOCUMENT RESUME

ED 117 165 TM 005 020

AUTHOR	Doerann-George, Judith
TITLE	The Effect on Non-Normal Distributions on the Integrated Moving Average Model of Time-Series Analysis.
PUB DATE	75
NOTE	17p.; Paper presented at the Annual Meeting of the American Educational Research Association (Washington, D.C., March 30-April 3, 1975)
EDRS PRICE	MF-$0.76 HC-$1.58 Plus Postage
DESCRIPTORS	*Hypothesis Testing; *Mathematical Models; Statistics; *Time
IDENTIFIERS	Distributions (Statistical); *Integrated Moving Average Model; Power (Statistics)

ABSTRACT

The Integrated Moving Average (IMA) model of time series, and the analysis of intervention effects based on it, assume random shocks which are normally distributed. To determine the robustness of the analysis to violations of this assumption, empirical sampling methods were employed. Samples were generated from three populations; normal, moderately non-normal, and severely non-normal. The samples were combined with values of other quantities in the model, the resulting "observations" subjected to time-series analysis, and the effect on empirical significance levels noted. The analysis of interventions based on the IMA model was robust to violations of the normality assumption. (Author)

THE EFFECT OF NON-NORMAL DISTRIBUTIONS ON THE INTEGRATED MOVING AVERAGE MODEL OF TIME-SERIES ANALYSIS

Judith Doerann-George
Indiana University

Abstract

The Integrated Moving Average (IMA) model of time series, and the analysis of intervention effects based on it, assume random shocks which are normally distributed. To determine the robustness of the analysis to violations of this assumption, empirical sampling methods were employed. Samples were generated from three populations; normal, moderately non-normal, and severely non-normal. The samples were combined with values of other quantities in the model, the resulting "observations" subjected to time-series analysis, and the effect on empirical significance levels noted.

The analysis of interventions based on the IMA model was robust to violations of the normality assumption.

AERA Convention
April 1975
Washington, D.C.

THE EFFECT OF NON-NORMAL DISTRIBUTIONS ON THE INTEGRATED MOVING AVERAGE MODEL OF TIME-SERIES ANALYSIS[1]

Judith Doerann-George[2]
Indiana University

Model building and testing have performed an important role in the development of theories, and through theories in the advancement of knowledge. A serious consideration in the use of models in research is the degree to which it is possible in the research situation to conform to the assumptions inherent in the model. If it can be determined that the outcome of statistical tests based on the model are not altered by departure from the conditions or assumptions, the model is said to be robust.

The Integrated Moving Average (IMA) model, and statistical methods for estimating intervention effects developed by Box and Tiao (1965), promise to be useful in time-series analysis. They are, however, based on assumptions of normality and homogeneity of variance of the random shocks affecting the system. This study was designed to determine the robustness of the IMA model and related analysis to non-normality of the shocks.

[1]The author wishes to acknowledge the contributions of Profs. Vernon L. Hendrix and Jon Morris, both of the University of Minnesota, in the conceptualization of this study, and of David Garrett, Indiana University, in computer programming.

[2]Inquiries concerning this paper may be directed to Judith Doerann-George, Education 253, Indiana University, Bloomington, Indiana 47401

In the time-series quasi-experiment, observations of a variable are made at equally spaced time intervals. It is desired to make inferences regarding an alteration in the series associated with the introduction of an event or treatment at some point in the series. The alteration may be either a change in level or in drift of the series. The Integrated Moving Average model and related analytic procedures can be used in the situation described, because they provide a statistical test for changes in level and in drift of the series, while allowing for the nonindependence of the observations. Furthermore, the IMA model allows the variable observed to be the property of a system imbedded in 'white noise' or subject to random shocks. These shocks may be absorbed in the system over time.

The expression in the IMA model for the n_1 pretreatment observations (z_t) is:

$$z_t = L + \gamma\mu(t-1) + \mu + \gamma\sum_{j=1}^{t-1} a_j + a_t. \tag{1}$$

For the n_2 observations following intervention the expression is:

$$z_t = L + \gamma\mu(t-1) + \mu + \gamma\Delta(t-n_1-1) + \Delta + \delta + \gamma\sum_{j=1}^{t-1} a_j + a_t , \tag{2}$$

where μ = the drift characteristic of the series
L = the initial level of the series when observations begin
Δ = the change in drift of the series, due to intervention
δ = the change in level of the series, also due to intervention
γ = the interdependence parameter, equivalent to 1-proportion of shocks carried over to the following observation

A given observation or data value is considered to be a linear function of the four parameters μ, Δ, L, and δ, and a value of a_t, a random normal variable with variance σ_a^2 (Tiao in Glass and Maguire, 1968). In order to determine the parameter values from the observations z_t of a system, the z_t's are first transformed for a given γ:

$$y_1 = z_1 \tag{3}$$

$$y_t = z_t - z_{t-1} + (1-\gamma)y_{t-1} \quad \text{for } t = 2, \ldots\ldots, n_1+n_2 \tag{4}$$

The vector of y's which results can be expressed by the linear model

$$y = X\beta + e \tag{5}$$

where X is the design matrix, β^{τ} the vector of parameters (μ, Δ, L, δ) and e the vector of values of the random normal variable a_t with variance σ_a^2.

Least-squares estimates of the four parameters are obtained, and (based on the assumption of normality of a_t and sampling theory) the following distributional statements can be made:

$$t \sim \frac{\hat{\mu} - \mu}{S_a\sqrt{C^{11}}} \qquad t \sim \frac{\hat{L} - L}{S_a\sqrt{C^{33}}}$$

$$t \sim \frac{\hat{\Delta} - \Delta}{S_a\sqrt{C^{22}}} \qquad t \sim \frac{\hat{\delta} - \delta}{S_a\sqrt{C^{44}}}$$

where C^{jj} is the j^{th} diagonal element of the matrix $(X^{\tau}X)^{-1}$.

The t-ratios (estimate/standard error) can be used to determine the probability of intervention effects, in the cases of δ and Δ. T-ratios are also calculated for L and μ.

Procedures

To test the robustness of the IMA model under violations of the normality assumption, it was necessary to obtain data that were non-normal and to subject such data to time-series analysis. The origin of the data is not important to the problem, so long as they conform to the characteristics needed (Hammersley and Handscomb, 1964). Consequently, simulation techniques were used to generate the random shock values a_t.

Three populations of random shocks were selected which had desired degrees of skewness and kurtosis; one normal, one moderately non-normal, and one severely non-normal. The binomial n, p parameters which characterize each of the three populations were determined by a recently developed technique (Martin and Hendrix, 1974). Each of the three n, p pairs thus determined was used with values from a random number generator to create 1000 samples of 60 each. The samples were standardized to mean zero and standard deviation one, before use in the time series. (Such standardization does not affect the value of skewness or kurtosis). The actual skewness and kurtosis of these samples compared well with the skewness and kurtosis as calculated from binomial values n and p. See Table 1.

TABLE 1: Comparison of Desired and Actual Measures of Skewness (β_1) and Kurtosis (β_2)

		Population	
	I	II	III
Binomial Input Value n	100	40	40
p	.5	.9870	.9958
Desired $\beta_1 = \frac{(q-p)^2}{npq}$	0.0000	1.8484	5.8775
$\beta_2 = \frac{1-6pq}{npq} + 3$	2.9800	4.7984	8.8275
Actual $\beta_1 = \frac{M_3^2}{M_2^3}$ *	0.0000	1.7641	5.6020
$\beta_2 = \frac{M_4}{M_2^2}$	2.9881	4.8082	8.6928
$*M_j = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^j}{n}$, n =	1, 2, ...	60,000	

Four values of γ, the interdependence parameter in time series, and three values of δ, μ, and Δ, other time-series parameters, were selected and varied systematically for each simulation run. The values of γ were 0.1, 0.5, 1.0, and 1.5, which adequately represent the range of γ values found in practice. For the other parameters, values of 0.5, 0.0, and

-0.5 were selected to provide tests of the null condition and to make it unnecessary to use supply distributions with positive and negative skewness. A few trial runs indicated to the investigator that 0.5 would be a reasonable magnitude of treatment effect. L, the initial level of the time series, was maintained at zero throughout the study.

Combinations of the four values of γ, and three each of δ, μ, and Δ yielded 108 parameter sets, and a different parameter set was used in each simulation run. A computer simulation run consisted of the generation of random shock samples from one population which were then combined with a set of time-series parameter values to create 'observations' of a time series. Each of 1000 sets of 60 observations were subjected to time-series analysis and the resulting t's tallied. The entire process was repeated with samples from the remaining two supply distributions.

More specifically, the 60 values in a sample from one of the three populations were combined with input values of γ, δ, μ, and Δ (L=0) according to the linear model of time series (equations 1 and 2) to yield 30 pre-intervention and 30 post-intervention data values or observations. The data set was analyzed, using a program based on one developed by Glass and Maguire (1968). From a data set and the true, or input, value of γ the following were computed: least-squares estimates of μ, Δ, L and δ, the standard error of each estimate, and four values of t obtained by dividing each estimate by its standard error. Consequently, each of the 108 simulation runs produced 12,000 t's, 1000 for each of four parameters and the three populations.

The 12 distributions of 1000 t's each were compared to the t-distribution with n-4=56 df to determine whether actual or empirical significance levels differed from nominal (.10, .05, and .01) ones. This was done by scanning the 1000 t's for a single supply distribution and a single parameter, and tallying the number of t's more extreme than the critical $t_{\alpha/2}$ values of ±1.671, ±2.000, and ±2.600. The entire process of data generation, parameter estimation, and tally of the 12 distributions of 1000 t's was executed with all of the 108 input parameter sets.

Results and Conclusion

Each of the 108 simulation runs yielded 36 empirical significance levels, one for each combination of the three populations, four estimated parameters, and three nominal significance levels. To condense those results, the results from similar input conditions were combined.

The four values of γ were used for 27 runs each. The -.5, 0.0, and +.5 values of μ, Δ, and δ were employed in the 27 possible combinations, once with each γ value. L, the initial series level, was maintained at zero for all 108 runs.

Consequently, with each γ value, results were obtained for 27 runs in which L was zero; there were nine when μ=-.5, nine when μ=0.0, and nine for which μ=+.5. Similarily, there were nine runs each in which the input values of Δ and of δ were -.5, 0.0, and +.5. To summarize overall trends, the empirical significance levels for a single parameter which were obtained with the same input value of that parameter were averaged, separately for each γ value.

When the null hypothesis was true for all or some of the parameters μ, Δ, L and δ (see Table 2, Appendix), the proportion of t-ratios more extreme than $t_{\alpha/2}$ (the critical t for two-tailed nominal alpha level α) was close to the nominal level of significance for all supply distributions, especially when $\gamma = .01$. At the higher values of γ, 0.5, 1.0, and 1.5, non-normality of the random shock supply distribution did seem to broaden the extreme tails of the t-distributions; at $\alpha = .01$, probability of Type I error usually increased with increasing non-normality of population, especially for L and δ. This effect was slight for μ, the series drift. Actual significance levels associated with zero input values of Δ, the change in drift, were not affected in any systematic way by non-normality of population.

When μ, Δ, and/or δ input values were ±.5 (see Tables 3 and 4, Appendix), nominal and actual significance levels were very similar when $\gamma = .01$, which indicates that non-zero drift (μ) or treatment effects (Δ and δ) would be difficult to detect regardless of population. At higher levels of γ, non-zero input values showed an effect in increased empirical significance level obtained; this is particularly marked for μ, moderate for Δ, and least severe for δ. (See Figure 1 for

γ = 1.0 and input values = -.5.) There was a tendency for actual significance levels to increase with non-normality of supply distribution. However observable trends were not consistent for all three parameters across γ levels or at all α levels. In all cases, any variation apparently associated with non-normality of supply distribution was small compared to the change in significance level due to non-zero input values of μ, Δ, and δ.

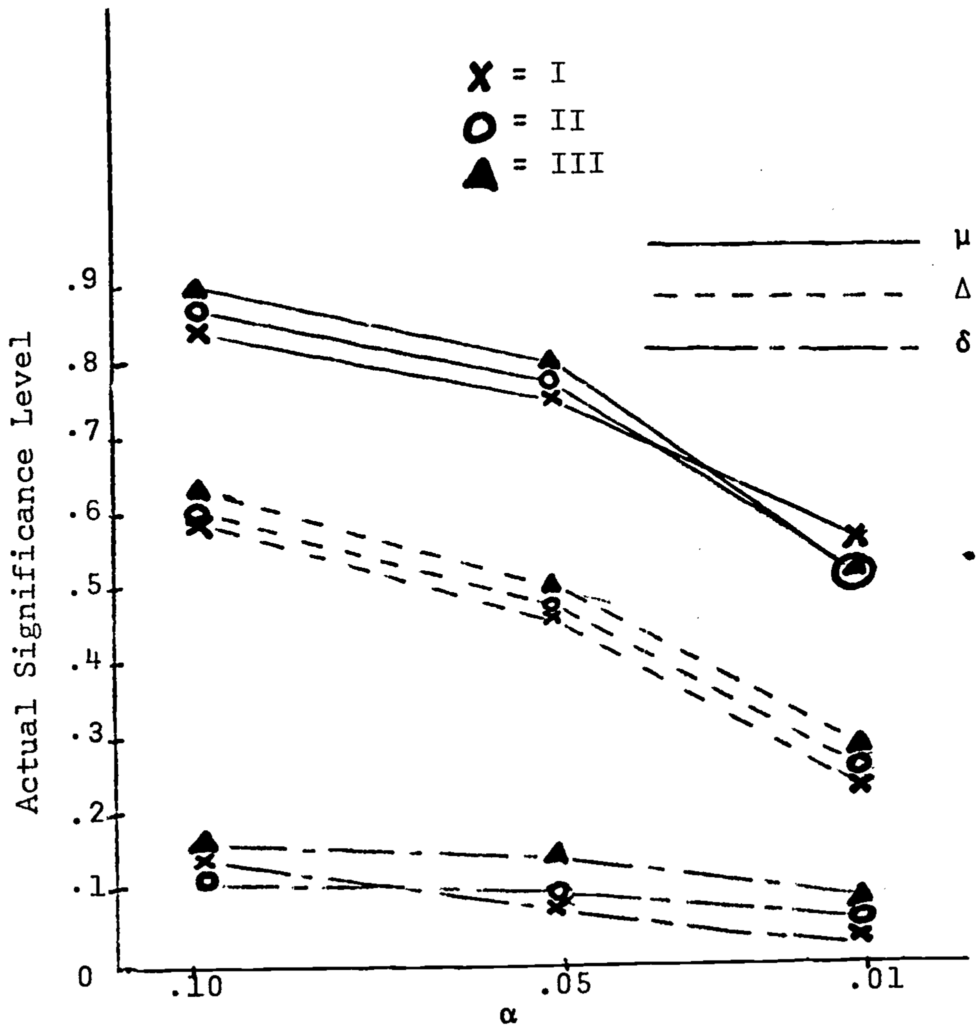

FIGURE 1: Mean Actual Significance Levels for Parameters μ, Δ, and δ by Alpha Level for Three Populations when γ = 1.0 and Input Values = -.5

It can be concluded that the analysis of time series based on the IMA model is robust to violations of the assumption of normality of the random shock population, particularly at alpha levels of .10 and .05. There does appear to be a slight increase in probability of Type I error with non-normality of shocks at α = .01. However, when the null hypothesis was not true, the probability of rejection of the hypothesis was not affected consistently by population type, and differences in rejection probability due to population type were negligible in comparison to actual significance level magnitude due to treatment effect (Δ,δ) or series drift (μ).

Variations apparently due to population type were not consistent enough nor of sufficient magnitude to warrant concern regarding random shock normality when setting alpha levels. In an experimental situation, of course, it would not be known whether the null hypothesis regarding Δ and δ is true, since those are possible treatment effects to be detected by the analysis. If there is sufficient data on the system being studied, the probable values of L and μ could be known, since those are characteristic of a given series. Consequently, it seems justifiable to recommend that possible non-normality of random shocks not be of primary concern to a researcher who is choosing alpha levels for a particular experiment employing the IMA model in time-series analysis.

REFERENCES

Box, G. E. P., & Tiao, G. C. A change in level of a nonstationary time series. Biometrika. 1965, 52, 181-192.

Glass, G. V., & Maguire, T. O. Analysis of time series quasi-experiments. Final report, U. S. Office of Education Bureau of Research, 1968.

Hammersley, J. M., & Handscomb, D. C. Monte Carlo Methods, London, Methuen and Co., Ltd., 1964.

Martin, G. R., & Hendrix, V. L. Two methods of generating statistical distributions whose first four moments are known. Paper presented at the annual meeting of the American Educational Research Association, Chicago, April, 1974.

TABLE 2: Empirical Significance Levels for Four Parameters, Averaged Over All Cases of Zero Input Value of Each Parameter

γ	Parameter	No. of Runs Averaged	Population	α = .10	α = .05	α = .01
.01	μ	9	I	.093	.050	.011
			II	.099	.048	.008
			III	.099	.049	.009
.01	Δ	9	I	.096	.050	.009
			II	.098	.046	.009
			III	.102	.048	.008
.01	L	27	I	.095	.050	.011
			II	.099	.049	.009
			III	.098	.051	.011
.01	δ	9	I	.100	.053	.010
			II	.095	.050	.009
			III	.100	.049	.009
.5	μ	9	I	.100	.051	.010
			II	.102	.052	.013
			III	.102	.050	.020
.5	Δ	9	I	.096	.052	.010
			II	.099	.052	.009
			III	.099	.044	.007
.5	L	27	I	.101	.049	.010
			II	.077	.049	.018
			III	.092	.061	.032
.5	δ	9	I	.103	.050	.008
			II	.072	.046	.017
			III	.091	.062	.032
1.0	μ	9	I	.105	.052	.009
			II	.104	.053	.013
			III	.114	.065	.017
1.0	Δ	9	I	.100	.053	.010
			II	.103	.054	.009
			III	.101	.049	.008
1.0	L	27	I	.103	.052	.014
			II	.081	.057	.021
			III	.132	.091	.036
1.0	δ	9	I	.105	.052	.010
			II	.089	.067	.024
			III	.131	.092	.038

TABLE 2 (Continued)

γ	Parameter	No. of Runs Averaged	Population	α = .10	α = .05	α = .01
1.5	μ	9	I	.105	.054	.010
			II	.109	.053	.012
			III	.110	.059	.018
1.5	Δ	9	I	.099	.052	.010
			II	.103	.054	.009
			III	.096	.045	.007
1.5	L	27	I	.103	.051	.010
			II	.096	.057	.019
			III	.125	.081	.029
1.5	δ	9	I	.101	.050	.009
			II	.096	.055	.018
			III	.126	.079	.029

TABLE 3: Empirical Significance Levels for Three Parameters, Averaged Over All Cases of -.5 Input Value of Each Parameter

γ	Parameter	No. of Runs Averaged	Population	α = .10	α = .05	α = .01
.01	μ	9	I	.108	.054	.012
			II	.108	.053	.009
			III	.109	.057	.013
.01	Δ	9	I	.101	.049	.008
			II	.106	.049	.012
			III	.105	.050	.009
.01	δ	9	I	.105	.050	.009
			II	.102	.050	.010
			III	.106	.054	.009
.5	μ	9	I	.828	.733	.490
			II	.843	.838	.473
			III	.872	.766	.471
.5	Δ	9	I	.549	.429	.218
			II	.571	.443	.227
			III	.583	.461	.249
.5	δ	9	I	.156	.086	.023
			II	.148	.100	.043
			III	.152	.115	.055
1.0	μ	9	I	.843	.754	.518
			II	.871	.772	.512
			III	.903	.795	.512
1.0	Δ	9	I	.594	.462	.235
			II	.599	.467	.250
			III	.617	.492	.274
1.0	δ	9	I	.142	.081	.020
			II	.115	.090	.049
			III	.157	.140	.079
1.5	μ	9	I	.857	.774	.541
			II	.877	.788	.527
			III	.915	.822	.538
1.5	Δ	9	I	.597	.465	.243
			II	.609	.483	.252
			III	.623	.499	.276
1.5	δ	9	I	.157	.090	.025
			II	.146	.097	.045
			III	.147	.125	.067

TABLE 4: Empirical Significance Levels for Three Parameters, Averaged Over All Cases of +.5 Input Value of Each Parameter

γ	Parameter	No. of Runs Averaged	Population	α = .10	α = .05	α = .01
.01	μ	9	I	.104	.053	.013
			II	.113	.059	.013
			III	.119	.058	.015
.01	Δ	9	I	.105	.051	.010
			II	.106	.052	.009
			III	.105	.053	.009
.01	δ	9	I	.099	.050	.010
			II	.104	.052	.011
			III	.106	.055	.011
.5	μ	9	I	.816	.715	.469
			II	.809	.723	.502
			III	.796	.714	.526
.5	Δ	9	I	.569	.449	.216
			II	.580	.453	.228
			III	.588	.470	.258
.5	δ	9	I	.149	.085	.033
			II	.138	.040	.009
			III	.067	.035	.018
1.0	μ	9	I	.849	.754	.517
			II	.819	.739	.534
			III	.811	.726	.535
1.0	Δ	9	I	.590	.467	.231
			II	.607	.480	.257
			III	.624	.496	.285
1.0	δ	9	I	.143	.078	.021
			II	.054	.025	.012
			III	.068	.036	.019
1.5	μ	9	I	.844	.758	.518
			II	.830	.748	.541
			III	.824	.743	.557
1.5	Δ	9	I	.610	.478	.242
			II	.610	.490	.253
			III	.621	.505	.285
1.5	δ	9	I	.156	.085	.022
			II	.146	.067	.014
			III	.149	.089	.023